Superposition and Conservation of Energy

Suppose you prepare a particle in a harmonic oscillator potential in a superposition of two energy stationary states, for example the $n = 0$ and the $n = 10$ states: $\Psi = \frac{1}{\sqrt{2}}(\psi_0 + \psi_{10})$. Now measure the energy of the particle. You will find either $E_0 = \frac{\hbar\omega}{2}$ or $E_{10} = \frac{21\hbar\omega}{2}$ with equal likelihood. In other words for an ensemble of identically prepared systems, one will find $\langle \hat{\mathcal{H}} \rangle = \frac{1}{2}(E_0 + E_{10}) = \frac{11}{2}\hbar\omega$. OK, but suppose you measure the particle once and find an energy of $\frac{21\hbar\omega}{2} > \frac{11}{2}\hbar\omega$. Where did the extra energy come from?

This illustrates the non-classical nature of a quantum superposition. First, note that the superposition state is NOT a determinate state for the Hamiltonian. It does not have a fixed value of energy. One is not justified in asking "where does the energy go/come-from?" upon measuring the energy of the particle prepared in a superposition of two or more energy eigenstates.

But what about energy conservation? It survives, but only in an ensemble average form. Consider the generalized Ehrenfest theorem:

$$\frac{d}{dt}\langle \hat{Q} \rangle = \frac{i}{\hbar}\langle [\hat{\mathcal{H}}, \hat{Q}] \rangle + \langle \frac{\partial \hat{Q}}{\partial t} \rangle \text{ with } \hat{Q} = \hat{\mathcal{H}}, \text{ then}$$

$$\frac{d}{dt}\langle \hat{\mathcal{H}} \rangle = \frac{i}{\hbar}\langle [\hat{\mathcal{H}}, \hat{\mathcal{H}}] \rangle + \langle \frac{\partial \hat{\mathcal{H}}}{\partial t} \rangle = 0 + 0 = 0$$

So "energy conservation" applies only to the Hamiltonian operator expectation value.